

Index Models

Topic 4
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Main Points

- The single-index model
- Regression and beta estimation



Single Factor Model

- The success of a portfolio selection rule depends on the quality of the input (estimates of expected security returns and the covariance matrix)
- If we have 50 stocks, using Markowitz portfolio selection model, we need 1325 estimates
 - 50 estimates of expected returns
 - 50 estimates of variances
 - $n(n-1) / 2 = 1225$ estimates of covariances



Single Factor Model

- Difficulty in applying Markowitz model
 - Requiring a huge number of estimates of covariances
 - Due to mutual inconsistency, errors exist in the estimation of correlation coefficients, leading to nonsensical results (i.e. negative variances)
- We need a model easier to implement

Single Factor Model

- We assume one macroeconomic indicator moves the security market as a whole
- We also assume all remaining uncertainty in stock returns is firm-specific

$$r_i = E(r_i) + m_i + e_i$$

- $E(r_i)$ is the expected return on the security as of the beginning of the holding period; m_i is the impact of unanticipated macro events on the security's return; e_i is the impact of unanticipated firm-specific events

Single Factor Model

- Since different firms have different sensitivities to macroeconomic events

$$r_i = E(R_i) + \beta_i F + e_i$$

β_i = sensitivity of a security's particular return to the factor

F = some macro factor commonly related to security returns

- When we use a broad market index like S&P 500 as the common factor, the model becomes 'single index model'

Single Index Model

$$(r_i - r_f) = \alpha_i + \beta_i(r_m - r_f) + e_i$$

↓
Risk Prem

↓
Market Risk Prem
or Index Risk Prem

- α_i = the stock's expected return if the market is neutral, i.e. if the market's excess return is zero
- $\beta_i(r_m - r_f)$ = the component of return due to movements in the market index
- e_i = firm specific component, not due to market movements

Risk Premium Format

Let: $R_i = (r_i - r_f)$ $\left\{ \begin{array}{l} \text{Risk premium} \\ \text{format, Why?} \end{array} \right.$
 $R_m = (r_m - r_f)$

$$R_i = \alpha_i + \beta_i(R_m) + e_i$$

Risk of Securities

- Since each security has two sources of risk-- market risk and firm-specific risk and covariance between the two is zero, we can break the variance of each stock into two components:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

- $\beta_i^2 \sigma_M^2$ the variance attributable to the uncertainty of the common macroeconomic factor
- $\sigma^2(e_i)$ the variance attributable to firm-specific uncertainty

Covariance of Securities

$$\text{Cov}(R_i, R_j) = \text{Cov}(\alpha_i + \beta_i R_M + e_i, \alpha_j + \beta_j R_M + e_j)$$

- Since alpha i and j are constants, their covariance with any variable is zero
- Firm-specific terms are assumed uncorrelated with the market and with each other
- So only source of covariance in the returns between the two stocks derives from their common dependence on the common factor

$$\text{Cov}(R_i, R_j) = \text{Cov}(\beta_i R_M, \beta_j R_M) = \beta_i \beta_j \sigma_M^2$$

Single Index Model

- We only need the following estimates for the input list for a single index model
 - N estimates of expected excess returns
 - N estimates of the sensitivity coefficients
 - N estimates of the firm-specific variances
 - 1 estimate of the variance of the common macroeconomic factor
- For a 50-security portfolio we only need 151 estimates rather than 1325

Single Index Model

- Advantages of a single index model
 - Reducing the number of inputs
 - Easier for security analysts to specialize
- Disadvantages of a single index model
 - Oversimplifying sources of real-world uncertainty

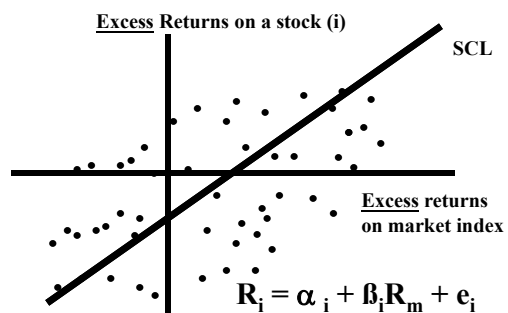
Estimating the Index Model

- We can look at the index model as a regression model since their assumptions are the same

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}$$

- Alpha: representing the average firm-specific excess return when the market's excess return is zero; intercept
- Beta: sensitivity of a stock to the market; the slope of the regression line
- e: firm specific effects; residual

Security Characteristic Line



Example

Month	GM return	Market return	monthly T-bill	Excess GM return	Excess Market return
1	6.06	7.89	0.65	5.41	7.24
2	-2.86	1.51	0.58	-3.44	0.93
3	-8.18	0.23	0.62	-8.80	-0.39
4	-7.36	-0.29	0.72	-8.08	-1.01
5	7.76	5.58	0.66	7.10	4.92
6	0.52	1.73	0.55	-0.03	1.18
7	-1.74	-0.21	0.62	-2.36	-0.83
8	-3.00	-0.36	0.55	-3.55	-0.91
9	-0.56	-3.58	0.60	-1.16	-4.18
10	-0.37	4.62	0.65	-1.02	3.97
11	6.93	6.85	0.61	6.32	6.24
12	3.08	4.55	0.65	2.43	3.90
Mean	0.023	2.377	0.62	-0.598	1.755
S.D.	4.970	3.329	0.047	4.969	3.317

Example

- In Excel, go to Tool – Data Analysis – Regression (If you use Office 97-2003)
- Choose 12 monthly GE excess returns as “Input Y range”, and 12 monthly market excess returns as “input X range”, then click “OK”
- If you include the first row in the data range, you need to choose “Label” before clicking “OK”

Example

- By running this regression, you are studying how much percentage market excess return can explain GE excess returns, what alpha is and what beta is
- Then Excel will give you a result table
- Part of the result table looks like the following

Example

SUMMARY OUTPUT

Regression Statistics

Multiple R	0.7582
R Square	0.5749
Adjusted R Square	0.5324
Standard Error	3.5492
Observations	12.0000

	Coefficients	Standard Error	t Stat	P-value
Intercept	-2.5923	1.1592	-2.2363	0.0493
Excess Market return	1.1362	0.3089	3.6777	0.0043

Example

- R-squared shows how much percentage the market return can explain the stock return; the higher the R-squared, the better your model
- Coefficients of “intercept” and “Excess Market Return” are Alpha and Beta
- T-test and P-value both tell you whether these two coefficients are statistically significant or not

Example 1

- As diversification increases, the firm-specific risk of a portfolio approaches

- _____.
- A. 0
 - B. 1
 - C. infinity
 - D. $n-1 * n$
 - E. none of the above



Example 2

- According to the index model, covariances among security pairs are
 - A. due to the influence of a single common factor represented by the market index return
 - B. extremely difficult to calculate
 - C. related to industry-specific events
 - D. usually positive
 - E. A and D



Example 3

- Analysts may use regression analysis to estimate the index model for a stock. When doing so, the slope of the regression line is an estimate of _____.
 - A. the α of the asset
 - B. the β of the asset
 - C. the σ of the asset
 - D. the δ of the asset
 - E. none of the above



Example 4

- In a factor model, the return on a stock in a particular period will be related to _____.
 - A. firm-specific events
 - B. macroeconomic events
 - C. the error term
 - D. both A and B
 - E. neither A nor B